

Neutrino proton elastic scattering and the spin structure of the proton

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Neutrino proton elastic scattering and polarized deep inelastic spin sum rules provide complementary information about the spin structure of the proton. We outline the two approaches and what they may teach us about the transition from current to constituent quarks in QCD.

1. Introduction

Understanding the spin structure of the proton is one of the most challenging problems facing subatomic physics: How is the spin of the proton built up out from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents ? What happens to spin in the transition between current and constituent quarks in low-energy QCD ? Key issues include the role of polarized glue and gluon topology in building up the spin of the proton.

Measurements of the proton's g_1 spin structure function in polarized deep inelastic scattering have been interpreted to imply a small value for the flavour-singlet axial-charge:

$$g_A^{(0)}|_{\text{pDIS}} = 0.15 - 0.35. \quad (1)$$

This result is particularly interesting [1] because $g_A^{(0)}$ is interpreted in the parton model as the fraction of the proton's spin which is carried by the intrinsic spin of its quark and antiquark constituents. The value (1) is about half the prediction of relativistic constituent quark models ($\sim 60\%$). It corresponds to a negative strange-quark polarization

$$\Delta s = -0.10 \pm 0.04 \quad (2)$$

(polarized in the opposite direction to the spin of the proton). The small value of $g_A^{(0)}|_{\text{pDIS}}$ extracted from polarized deep inelastic scattering has inspired vast experimental and theoretical activity to understand the spin structure of the proton (and related connections to the axial U(1) problem). New experiments are underway or being planned to map out the proton's spin-flavour structure and to measure the amount of spin carried by polarized gluons in the polarized proton – for a review see [2].

In this paper we explain the spin sum rule connecting g_1 and $g_A^{(0)}$ and how additional valuable information could be obtained from a precision measurement of neutrino proton elastic scattering.

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2. g_1 spin sum rules in polarized deep inelastic scattering

Sum rules for polarized deep inelastic scattering are derived starting from the dispersion relation for photon nucleon scattering:

$$A_1(Q^2, \nu) = \beta_1(Q^2) + \frac{2}{\pi} \int_{Q^2/2M}^{\infty} \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \text{Im} A_1(Q^2, \nu') \quad (3)$$

Here A_1 is the first form-factor in the spin dependent part of the forward Compton amplitude which is related to the g_1 spin structure function via $g_1 = \pi \frac{M}{\nu} A_1$ and $\beta_1(Q^2)$ denotes a possible subtraction constant (“subtraction at infinity”) from the circle when we close the contour in the complex plane [3]. (M denotes the proton mass.)

The value of $g_A^{(0)}$ extracted from polarized deep inelastic scattering is obtained as follows. One applies the operator product expansion and finds that the first moment of the structure function g_1 is related to the scale-invariant axial charges of the target nucleon:

$$\begin{aligned} \int_0^1 dx g_1^p(x, Q^2) &= \left(\frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_{\text{NS}\ell} \alpha_s^\ell(Q) \right\} \\ &+ \frac{1}{9} g_A^{(0)}|_{\text{inv}} \left\{ 1 + \sum_{\ell \geq 1} c_{\text{S}\ell} \alpha_s^\ell(Q) \right\} + \mathcal{O}\left(\frac{1}{Q^2}\right) - \beta_1(Q^2) \frac{Q^2}{4M^2} \end{aligned} \quad (4)$$

Here $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}|_{\text{inv}}$ are the isovector, SU(3) octet and scale-invariant flavour-singlet axial charges respectively. The Wilson coefficients $c_{\text{NS}\ell}$ and $c_{\text{S}\ell}$ are calculable in ℓ -loop perturbative QCD [4]. One then assumes no twist-two subtraction constant ($\beta_1(Q^2) = \mathcal{O}(1/Q^4)$) so that the axial charge contributions saturate the first moment at leading twist.

The first moment of g_1 is constrained by low energy weak interactions. For proton states $|p, s\rangle$ with momentum p_μ and spin s_μ

$$\begin{aligned} 2Ms_\mu g_A^{(3)} &= \langle p, s | (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) | p, s \rangle \\ 2Ms_\mu g_A^{(8)} &= \langle p, s | (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2\bar{s} \gamma_\mu \gamma_5 s) | p, s \rangle \end{aligned} \quad (5)$$

Here $g_A^{(3)} = 1.267 \pm 0.004$ is the isovector axial charge measured in neutron beta-decay; $g_A^{(8)} = 0.58 \pm 0.03$ is the octet charge measured independently in hyperon beta decays (using SU(3)) [5].

The scale-invariant flavour-singlet axial charge $g_A^{(0)}|_{\text{inv}}$ is defined by

$$2Ms_\mu g_A^{(0)}|_{\text{inv}} = \langle p, s | E(\alpha_s) J_{\mu 5} | p, s \rangle \quad (6)$$

where $J_{\mu 5} = (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s)$ is the gauge-invariantly renormalized singlet axial-vector operator and $E(\alpha_s) = \exp \int_0^{\alpha_s} d\tilde{\alpha}_s \gamma(\tilde{\alpha}_s) / \beta(\tilde{\alpha}_s)$ is a renormalization group factor which corrects for the two-loop anomalous dimension $\gamma(\alpha_s)$ of $J_{\mu 5}$ and which goes to one in the limit $Q^2 \rightarrow \infty$.

In the isovector channel The Bjorken sum rule [6] for the isovector part of g_1 , $\int_0^1 dx (g_1^p - g_1^n)$, has been confirmed in polarized deep inelastic scattering experiments at the level of 10%. Substituting the values of $g_A^{(3)}$ and $g_A^{(8)}$ from beta-decays (and assuming no subtraction constant correction) in the first moment equation (4) polarized deep inelastic data

implies $g_A^{(0)}|_{\text{pDIS}} = 0.15 - 0.35$ for the flavour singlet moment. The small x extrapolation of g_1 data is the largest source of experimental error on measurements of the nucleon's axial charges from deep inelastic scattering.

QCD theoretical analysis leads to the formula [7, 8, 9]:

$$g_A^{(0)} = \left(\sum_q \Delta q - 3 \frac{\alpha_s}{2\pi} \Delta g \right)_{\text{partons}} + \mathcal{C}_\infty \quad (7)$$

where $g_A^{(0)} = g_A^{(0)}|_{\text{inv}}/E(\alpha_s)$. Here $\Delta g_{\text{partons}}$ is the amount of spin carried by polarized gluon partons in the polarized proton and $\Delta q_{\text{partons}}$ measures the spin carried by quarks and antiquarks carrying “soft” transverse momentum $k_t^2 \sim m^2, P^2$ where m is the light quark mass and P is a typical gluon virtuality; \mathcal{C}_∞ denotes a non-perturbative gluon topological contribution which has support only at Bjorken x equal to zero (so that it cannot be measured in polarized deep inelastic scattering) [9]. Since $\Delta g \sim 1/\alpha_s$ under QCD evolution, the polarized gluon term $[-\frac{\alpha_s}{2\pi} \Delta g]$ in Eq.(7) scales as $Q^2 \rightarrow \infty$ [7]. The polarized gluon contribution corresponds to two-quark-jet events carrying large transverse momentum $k_t \sim Q$ in the final state from photon-gluon fusion [8].

The topological term \mathcal{C}_∞ may be identified with a leading twist “subtraction at infinity” in the original dispersion relation (3), whence $g_A^{(0)}|_{\text{pDIS}}$ is identified with $g_A^{(0)} - \mathcal{C}_\infty$ [3]. It probes the role of gluon topology in dynamical axial U(1) symmetry breaking in the transition from current to constituent quarks in low energy QCD. The deep inelastic measurement of $g_A^{(0)}$, Eq.(1), is not necessarily inconsistent with the constituent quark model prediction 0.6 *if* a substantial fraction of the spin of the constituent quark is associated with gluon topology in the transition from constituent to current quarks (measured in polarized deep inelastic scattering).²

Understanding the transverse momentum dependence of the contributions to (7) is essential to ensure that the theory and experimental acceptance are correctly matched when extracting information from semi-inclusive measurements about the individual valence, sea and gluonic contributions [10]. Recent semi-inclusive measurements [11] using a forward detector and limited acceptance at large transverse momentum ($k_t \sim Q$) exhibit no evidence for the large negative polarized strangeness polarization extracted from inclusive data and may, perhaps, be more comparable with $\Delta q_{\text{partons}}$ than the inclusive measurement (2), which has the polarized gluon contribution included. Further semi-inclusive measurements with increased luminosity and a 4π detector would be valuable.

A direct measurement of the strange quark axial charge could be made using neutrino proton elastic scattering.

3. νp elastic scattering

Neutrino proton elastic scattering measures the proton's weak axial charge $g_A^{(Z)}$ through elastic Z^0 exchange. Because of anomaly cancellation in the Standard Model the weak neutral current couples to the combination $u - d + c - s + t - b$, *viz.*

$$J_{\mu 5}^Z = \frac{1}{2} \left\{ \sum_{q=u,c,t} - \sum_{q=d,s,b} \right\} \bar{q} \gamma_\mu \gamma_5 q \quad (8)$$

²The electroweak version of this physics can lead to the formation of a “topological condensate” in the early Universe in parallel with the generation of the baryon number asymmetry [12].

It measures the combination

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s) + (\Delta c - \Delta b + \Delta t) \quad (9)$$

where

$$2Ms_\mu\Delta q = \langle p, s | \left(\bar{q}\gamma_\mu\gamma_5 q \right) | p, s \rangle \quad (10)$$

Heavy quark renormalization group arguments can be used to calculate the heavy t , b and c quark contributions to $g_A^{(Z)}$. Working to NLO and expressing the result in terms of just renormalization scale invariant quantities one finds the result [13]

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s)_{\text{inv}} + \mathcal{H}(\Delta u + \Delta d + \Delta s)_{\text{inv}} + O(m_{t,b,c}^{-1}) \quad (11)$$

where \mathcal{H} is a polynomial in the running couplings $\tilde{\alpha}_h$,

$$\mathcal{H} = \frac{6}{23\pi}(\tilde{\alpha}_b - \tilde{\alpha}_t) \left\{ 1 + \frac{125663}{82800\pi}\tilde{\alpha}_b + \frac{6167}{3312\pi}\tilde{\alpha}_t - \frac{22}{75\pi}\tilde{\alpha}_c \right\} - \frac{6}{27\pi}\tilde{\alpha}_c - \frac{181}{648\pi^2}\tilde{\alpha}_c^2 + O(\tilde{\alpha}_{t,b,c}^3) \quad (12)$$

Here $(\Delta q)_{\text{inv}}$ denotes the scale-invariant version of Δq and $\tilde{\alpha}_h$ denotes Witten's renormalization group invariant running couplings for heavy quark physics. Taking $\tilde{\alpha}_t = 0.1$, $\tilde{\alpha}_b = 0.2$ and $\tilde{\alpha}_c = 0.35$ in (12), one finds a small heavy-quark correction factor $\mathcal{H} = -0.02$, with LO terms dominant.

Modulo the small heavy-quark corrections quoted above, a precision measurement of $g_A^{(Z)}$, together with $g_A^{(3)}$ would provide a rigorous weak interaction determination of $(\Delta s)_{\text{inv}}$. The axial charge measured in νp elastic scattering is independent of any assumptions about the presence or absence of a ‘‘subtraction at infinity’’ correction to the first moment of g_1 and the $x \sim 0$ behaviour of g_1 .

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